CSE 512/CS 554 Homework Assignment 3

Due April 17, 2008

1. For a given problem dimension n, what is the optimal number of processors p for dense matrix-vector multiplication

   (a) using a 1-D row or column algorithm on a 1-D processor mesh?

   (b) using a 2-D algorithm on a 2-D processor mesh?

   Your answers should be an expression for p as a function of n.

2. For LU factorization of an $n \times n$ matrix using a given number of processors p, what is the tradeoff point in the dimension n such that a 1-D row or column algorithm on a 1-D processor mesh is best for matrices of order less than n, and a 2-D algorithm on a 2-D mesh is best for matrices of order greater than n?

3. Using $cmod(j, k)$ and $cdiv(j)$ as tasks, draw a task graph (with tasks as nodes and data dependences as edges) for column-oriented Cholesky factorization of a dense symmetric positive definite matrix of order 5.

4. (a) Draw the sparsity pattern of a matrix of order 21 corresponding to a 3×7 rectangular grid numbered in the "natural" (say, column-wise) order, using $\times$ for nonzero and blank for zero entries.

   (b) Draw the sparsity pattern of the Cholesky factor of the matrix, indicating original nonzeros by $\times$ and fill by $+$.

   (c) Draw the elimination tree for the matrix.

   (d) What is the maximum number of *cdiv* operations that can be done simultaneously?

5. Repeat parts (a-d) of the previous problem using a nested dissection ordering.

6. (a) Formulate a 1-D row fan-out parallel algorithm for solving an *upper* triangular linear system $\boldsymbol{U}\boldsymbol{x} = \boldsymbol{b}$.

   (b) Formulate a 1-D column fan-in parallel algorithm for solving an *upper* triangular linear system $\boldsymbol{U}\boldsymbol{x} = \boldsymbol{b}$.

7. (a) In the 1-D column cyclic algorithm for solving triangular systems, how long does it take for the segment to return to a given processor, assuming that each intervening processor is ready to process the segment as soon as it arrives? Use the standard model for computation and communication.

   (b) How many component updates can be computed by the processor during this time?